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Quantifying the Impact of 8Ps Learning Model on Grade 12 Students' Performance in Stationary Points Differential Calculus

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Abstract

Differential calculus is a crucial topic in mathematics that helps students hone critical thinking and problem-solving skills. Nonetheless, existing literature reports that grade 12 students frequently encounter challenges learning stationary points, a central concept in differential calculus, owing to the limited effectiveness of traditional teaching methods. To address this concern, this study introduced the 8Ps learning model and examined its impact on student performance in stationary points differential calculus. Grounded in Dewey's theory of reflective inquiry and experiential learning, this study employed quasi-experimental design of pre-test/post-test, non-equivalent control group. The sample comprised 253 grade 12 students and 8 teachers – 128 students and 4 teachers as experimental group and, 125 students and 4 teachers as control group. A mathematics achievement test was pre- and post-administered to both groups for data to evaluate whether any significant mean differences existed between the experimental group exposed to the 8Ps treatment and the control group taught through traditional methods. Data analysis through descriptive statistics and independent samples t-test yielded significant statistical improvement within the 8Ps group relative to the control group. Findings suggest the prospect of the 8Ps learning model to enhance student performance in stationary points differential calculus. The study recommends incorporating the model into calculus education to increase students' mathematics learning gains.

Keywords: *Quantitative Impact, 8Ps Learning Model, Stationary Points Differential Calculus, Student Performance, Mathematics Education.*

Introduction

This quantitative inquiry probes the impact of the 8Ps learning model on the grade 12 students' performance in stationary points differential calculus. In the educational curricula of most nations, students are introduced to differential calculus during the 11th or 12th grade, typically between ages 16 and 18. This initial exposure lays the groundwork for more advanced calculus concepts that they will encounter in higher education (Haghjoo & Reyhani, 2021; Nortvedt & Siqveland, 2019; Thompson & Harel, 2021). At this upper high school level, the curriculum emphasizes core foundational topics such as limits, derivatives, differentiation rules, tangents, optimization problems, motion analysis, graphical interpretations, and first principles (Dreyfus, 2021). Gaining proficiency in these basic concepts is crucial for students to comprehend the mechanics of differential calculus and its widespread applications in scientific and engineering fields (Haghjoo & Reyhani, 2021).

In South Africa, the context for this study, students encounter differential calculus for the first time in grade 12. This concept has been an integral part of the grade 12 mathematics curriculum for years (DoE, 2007, 2012; Pillay, et al., 2014). As outlined in the Curriculum Assessment Policy Statement (CAPS), which governs educational practices from grade R to 12, the students are expected to be able to understand and apply differential calculus principles to assess the rate of change in basic non-linear functions; draw graphs of cubic and other relevant polynomial functions using differentiation to identify stationary points (maxima, minima, and points of inflection), and employ the factor theorem alongside other techniques to find x-axis

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intercepts. Besides, they should be capable of solving practical problems related to optimization and rates of change (DBE, 2003, 2012).

Differential calculus examines how one quantity changes in relation to another by determining derivatives that reflect a function's sensitivity to changes in its input variables (Charles-Ogan & Ibibo, 2018). The derivative serves as a basic tool used in differential calculus to indicate how much a function changes at a specific point. Understanding stationary points – where a function's derivative is zero or undefined – is vital for developing students' understanding of mathematical principles and problem-solving skills (Kadry, 2024; Thomas, 1996).). For grade 12 students preparing for higher education, mastering this concept becomes imperative owing to its broad real-world applications particularly across disciplines like physics, engineering and economics (Bedada, 2021; Nortvedt & Siqveland, 2018). Apart from considering its wide applications, this study's focus on it was further reinforced by the fact that it accounts for 35 ± 3 of the total 150 marks allotted to the National School Certificate (NSC) mathematics paper 1 (DBE, 2023).

Nevertheless, grade 12 students globally, including those in South Africa, struggle to understand differential calculus, particularly stationary points (Dreyfus, 2021; Haghjoo & Reyhani, 2021; Mendezabal, & Tindowen, 2018). As established in research, the traditional teaching methods have not fully engaged students or clarified this complex concept, leading to insufficient understanding and retention (Bedada, 2021; Yimer, 2022). Associated with students passively absorbing information from the teacher, rote learning and isolated study of differential calculus concepts and procedures, the traditional teaching methods have produced low academic achievement and limited knowledge transfer (Mendezabal, & Tindowen, 2018; Yimer, 2022). Thus, students who are products of the approach do not find it easy to connect their differential calculus knowledge meaningfully to practical situations (Panero, 2024; Vacalares, et al., 2024).

In a challenging domain like differential calculus, the effectiveness of instructional strategies plays a pivotal role in student performance (Benderbal & Djedoui, 2024; Nuñez et al., 2023; Simovwe, 2020). For an instructional strategy to enhance mathematics instruction and student performance, it must incorporate key components such as: actively engaging students in mathematical tasks that encourage critical thinking – analysis, synthesis and evaluation (Abd-Hamid, et al., 2019; Purnomo, et al., 2024); adapting to diverse learning styles and needs (Vacalares, et al., 2024); using formative assessments regularly to track and adjust teaching techniques based on student progress (Wiliam, 2020); supporting peer collaboration (Miquel & Duran, 2017), and linking mathematical concepts to practical, real-world contexts (Abramovich, et al., 2019).

With the prevailing current shift in educational paradigms towards prioritizing interactive, student-focused learning, there is an increasing desire for instructional approaches that are capable of productively cultivating deep mathematics learning experience, particularly in differential calculus (Bedada, 2021; Sebsibe, 2019; Simovwe, 2020). In response to this need, we developed the 8Ps learning model, a mathematical problem-solving model that emphasizes active student involvement and structured learning. This model prioritizes a practical, individualized, and interactive learning experience for students. The primary goal is to seek to measure the effect of using it on student performance, especially in stationary points differential calculus. The next section explains how the 8Ps learning model was constructed for this study.

Constructing the 8Ps Learning Model

The need to improve mathematics education has prompted the development of various learning models that offer a range of strategies for mathematical problem solving. Prominent examples include frameworks by Pólya (1945), Mason, et al. (1982), Burton (1984), Schoenfeld (1985), Mason, et al. (2010), Cherry (2011), Kirkley (2003), Wilson, et al. (1993), Maccini and Gagnon (2006), and Faucette and Pittman (2015), each with its distinct strengths and limitations. Recent studies have continued to explore these learning models, highlighting their relevance in contemporary educational contexts (Brijlall, 2015). As part of the myriad of interventions, this study introduces the 8Ps learning model by drawing on Pólya's learning framework, which has significantly influenced subsequent problem-solving models.

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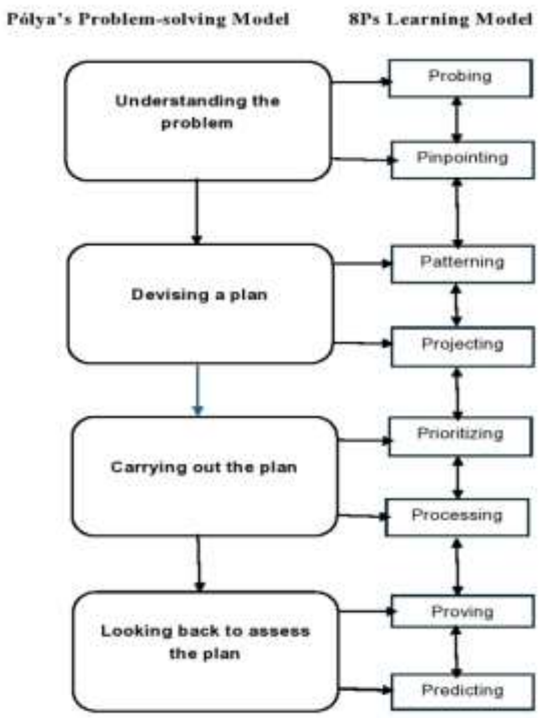
Figure 1

The Construction of the 8Ps Learning Model as Inspired by Pólya (1945)



Pólya's 1945 seminal work, *How to Solve It*, outlines four essential stages for problem solving*: understanding the problem, devising a plan, carrying out the plan* and *looking back*. Presenting a structured and efficient approach for mathematical problem solving, these four steps highlight the need for thorough comprehension, strategic planning, careful implementation and critical evaluation as core elements. The intent is to guide students through discovery and invention, helping them generate new ideas and knowledge in the problem-solving process (Chacón-Castro, et al., 2023). The 8Ps learning model expands on these four steps, including four additional ones.

Pólya's first step, *understanding the problem*, requires students to clearly define the problem, identifying relevant mathematical concepts and procedures. Pólya suggests determining what is known, what is unknown and the given conditions It further suggests rephrasing the problem and possibly representing it with visual aids such as sketches or diagrams. The 8Ps learning model builds on this by splitting the stage into two*: probing* and *pinpointing*. These two phases emphasize a comprehensive understanding of the problem to avoid errors in subsequent steps. *Probing* involves examining the problem closely to determine its demands, akin to the

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entry phase in the seven-step problem-solving process by Mason, et al. (1982/2010). *Pinpointing* focuses on identifying the key elements and conditions of the problem. It is likeable to *searching the problem* suggested by Maccini and Gagnon (2006) and *identifying the problem* by Cherry (2011).

Pólya's second step, *devising a plan*, is about selecting strategies such as working backwards, finding patterns, and relating the current problem to similar ones earlier solved. The plan can also involve breaking the problem into manageable parts or choosing an approach or a combination of approaches that appears most promising. The 8Ps model extends this to two phases: *patterning* and *projecting*. *Patterning* entails creating mathematical representations like charts, tables, maps, pictures or diagrams from given problems. Kirkley (2003) describes this phase as *representing the problem,* while Maccini and Gagnon (2006) explain it as *translating the problem*. *Projecting* centers on developing and regulating solution plans by selecting appropriate strategies, making proper decisions about the mathematical operations, assumptions and procedures to deploy. It is a stage for explicit plan and regulation of the solution process. Selecting the right strategies, Chacón-Castro, et al. (2023) remark, is vital for obtaining a logical solution and keeping the student engaged and motivated to learn more of the concept/subject.

His third step, *carrying out the plan*, requires implementing the developed strategy with accuracy and persistence. While carrying out calculations, constructions and other necessary tasks, the problem solver is expected to carefully and systematically follow the chosen strategy; keep track of the problem-solving process and make necessary adjustments when going wrong and be persistent with the plan. In the 8Ps model, this stage is represented as *prioritizing* and *processing*. *Prioritizing* demands trimming down solution plans to select the most relevant ones. The tools to use to solve the problem and the order of applying them must be strategically determined. *Processing* is about implementing the chosen plan. For a reasonable solution, all the thoughts, patterns and selected solution ideas must be consciously executed.

Pólya's last step, *looking back,* is meant for reviewing and assessing the solution to ascertain correctness. The solution process must be reflected on to ensure no errors are made in the calculations, to understand the reasons for the logicality of the solution, and for generalizability or applicability of the solution to similar problems. The 8Ps model captures this final reflective step as its last two phases: *proving* and *predicting*. Proving assesses and justifies the solution, a way to ascertain that it makes sense. *Predicting* considers broader acceptability and possible generalizability of the solution. According to Burton (1984), this phase is *extension* of the solution.

Like Pólya's, the 8Ps learning model highlights strategic thinking and reflection during problem solving, making it a holistic approach rather than one solely aimed at obtaining the correct answer. Although we suggest following the eight phases orderly, we accept that not every phase is relevant to all problems, thereby allowing for adaptability in the problem-solving process. For example, a problem may not be easily illustrated through patterns. Again, moving back and forth along the model's phases is possible, an opportunity for the problem-solver who is confused at any phase to seek help from the previous phase(s). For instance, the student unsure of the most applicable strategy to execute (at *processing*) can revert to the previous three phases for clarity. This may also warrant revisiting *projecting* to be sure that the solution plans are properly pruned down, and/or to *prioritizing* to double-check that the plans are strategically selected. Furthermore, rechecking *processing* when in doubt of the logicality of the solution (at *proving*) may be advisable. All along, the teacher's role is essentially to guide and facilitate the problem-solving process while the students navigate the eight phases, promoting collaboration, discussion and peer learning.

While the 8Ps model initially seems complex with its eight phases, its multiple pathways to problem solving may enhance active learning and knowledge construction. By expanding on Pólya's foundational ideas, this model breaks down the problem-solving process into more detailed and specific steps, allowing for deeper engagement and adaptability to various learning styles while emphasizing cognitive processes. Additionally, the last phase of the 8Ps model, *predicting*, encourages students to use the solution (i.e. current understanding) to predict solutions to similar problems, thus taking this a step further than Pólya's final step, *looking back,* which primarily reflects on past solution strategies taken without explicitly promoting future predictions.

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Research Purpose

The purpose of this investigation is to assess the quantitative impact of the 8Ps learning model on grade 12 students' performance in the concept of stationary points in differential calculus.

Research Question and Hypothesis

To attain the stated purpose, the study sought to answer the question below:

Will the implementation of 8Ps learning model in teaching and learning stationary points in differential calculus have any statistically significant quantitative effect on grade 12 students' performance?

In accordance with the question, the study formulated the following hypothesis:

There is no statistically significant quantitative difference between the academic performances of the participants in the experimental group exposed to the 8Ps instruction and their counterparts in the control group taught with the traditional methods.

Theoretical Perspectives

This study derives its theoretical strength from John Dewey's theory of reflective thinking. Dewey, a major 20th Century American philosopher and educator, first presented the core principles of this theory in his 1910 publication, *How We Think*, and further elaborated on them in his 1938 work, *Logic: The Theory of Enquiry*. Dewey recommended a structured cognitive problem-solving process comprising five mental stages: gaining awareness of a problem; determining its nature; gathering relevant information; forming and testing a hypothesis and, lastly, making conclusions. This approach underpins educational practices which value critical thinking and problem-solving skills, urging students to engage more deeply in their learning. Tomlinson (1997) notes that, in contrast to Edward Thorndike's focus on behavior alone in problem solving, Dewey stressed the essentiality of both behavioral responses and the capacity for conscious thought.

Greenberger (2020) stresses that reflective thinking is a key component of effective learning, arguing that it should lead to action. He agrees to Dewey's view that reflection on one's experiences should go beyond understanding, by actively applying that understanding to real-world situations, thus making the learning process dynamic and action-driven. Similarly, Dimova and Kamarska (2015) express that Dewey's five stages of reflective thinking offer a coherent framework which enables students to integrate their experiences with their thoughts, a vital connection for deepening understanding and developing critical thinking skills in contemporary education. Popova (2014) explores Dewey's assertion that reflective thinking starts with a sense of discomfort, prompting reflection and desire for knowledge; that the thinking process must follow a structured sequence, with each phase contributing significantly towards problem solving; that linking these phases is crucial for students to validate their ideas through evidence-based reasoning, ultimately improving their problem-solving skills; that the thinking process must embrace uncertainty by rigorously assessing every idea before accepting it; and that reflective thought should be trained and protected against reasoning errors.

The 8Ps learning model aligns closely with Dewey's five mental stages. Its first phase, probing, introduced participants to identification of mathematical problems. This initial interaction with the problem, Greenberger (2020) remarks, is foundational in experiential learning and lays the groundwork for the enquiry process that Dewey's theory stresses. The second and third phases, pinpointing and patterning, guided participants in analyzing and diagnosing problems. Dimova and Kamarska (2015) underscore the cruciality of clearly determining the nature of a problem as a necessary part of reflexive inquiry. During the fourth and fifth phases, projecting and prioritizing, participants were taught to collect relevant information and formulate the preliminary hypotheses. Greenberger (2020) supports the notion that gathering

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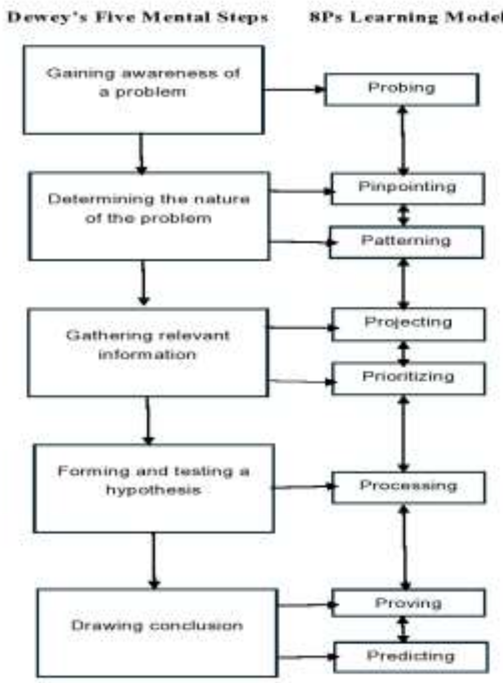
information through active engagement and collaboration enhances the learning process. In the sixth phase, processing, participants tested the hypotheses to decide their validity. Dimova and Kamarska (2015) confirm that evidence-based hypothesis testing is required for success in problem solving. In the last two phases, proving and predicting, participants learned to draw and assess conclusions. Greenberger (2020) asserts that reflecting on the findings and evaluating their effectiveness reinforces the learning outcomes. Overall, Dewey's philosophical stance of reflective thinking and active engagement is at the core of the 8Ps learning model.

Figure 2

The 8Ps Learning Model Gaining Theoretical Basis from Dewey's Five Mental Steps



Reviewed Literature

This study set out to quantify the impact of using the 8Ps learning model on grade 12 students' performance in stationary point differential calculus. The effectiveness of learning models in enhancing student performance has been a central theme in educational research, especially in mathematics education (Benderbal & Djedoui, 2024). A structured model (like the 8Ps learning model), with its focus on the principles of strategic thinking and reflective practices, seems promising to facilitate deep learning experiences, if effectively implemented (Shurygin et al., 2024). This literature review delves into past related studies that evaluate the impact of different learning models on student performance in differential calculus concepts, using the 8Ps learning model as a reference point. In so doing, it identifies research gaps and underscores the ability of the current study to contribute to pedagogical practices aimed at improving

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academic performance among grade 12 mathematics students. It also discusses the findings of the related past studies and their implications for the current study.

A 2017 study carried out in South Africa by Dlamini, et al. measured student performance in cubic functions within differential calculus. It considered a qualitative approach involving written assessments and interviews to identify students' misconceptions and conceptual difficulties in the concept. The study found that students usually struggled with connecting graphical representations of cubic functions to algebraic derivatives. The study recommended incorporating more interactive visual tools to improve conceptual understanding. Findings from Dlamini, et al. (2017) support the need to emphasize visual and graph-oriented activities obtainable within the 8Ps learning model to address common misconceptions of mathematical concepts.

In a qualitative study conducted in the United States, Mkhatshwa (2024) explored best practices for teaching derivatives by engaging 10 experienced calculus instructors through a structured 12-item questionnaire. The data collected were analyzed thematically, revealing several effective teaching strategies. Key findings included the importance of providing ample examples and practice problems, utilizing graphing utilities such as Desmos and GeoGebra, incorporating real-world applications to illustrate derivatives, and employing problem-solving strategies. The implications for the current study are significant. The innovative problem-solving strategies within the 8Ps learning model align well with the use of numerous real-world examples, potentially enhancing student engagement and comprehension of complex calculus concepts like stationary points. Furthermore, this study could contribute to developing a robust pedagogical framework within the 8Ps model that emphasizes effective teaching strategies for challenging calculus topics.

Nuñez et al. (2023) executed quasi-experimental research in the Philippines, with 46 grade 11 students to evaluate how effective the Enhancing Mastery & Expertise in Mathematics module (EM&EM, an innovative researcher-developed teaching module) was in boosting basic calculus performance. The study compared the experimental group using the EM&EM module to a control group that experienced the standard Department of Education (DepEd) module. Analysis of the independent samples and paired t-tests on both groups' pre-test, formative test and post-test marks produced statistically significant mean score differences, indicating that the EM&EM module effectively promotes student engagement and learning. These findings suggest that future research, like the present one proposing the 8Ps learning model, should concentrate on challenging mathematical concepts, innovative pedagogical practices, intensive problem-solving strategies, and stimulating learning environments.

In a case study involving 3 high school teachers in Zambia, Simovwe (2020) assessed the instructional practices in calculus. The study involved qualitative data collection through classroom observations, interviews and instructional materials review. Findings showed that inconsistent instructional approaches and limited use of interactive tools contributed to students' weak conceptual understanding of the calculus. Recommendations included enhancing teacher training and integrating more dynamic teaching strategies. The 8Ps learning model can address these issues by fostering interactive learning environments and providing structured teacher-guided activities to improve students' understanding of stationary points.

Sebsibe (2020) examined the challenges faced by grade 12 students in Ethiopia when learning calculus concepts and proposed strategies to address the difficulties. Utilizing mixed methods, the study involved quantitative surveys and qualitative interviews, revealing common issues such as misconceptions, anxiety regarding calculus, and ineffective teaching methods. Findings suggest that targeted interventions, including customized instructional strategies and supportive learning environments, could alleviate these challenges. Hence, the need for the present study as its 8Ps learning model is designed to cater to these specific student needs.

Vacalares, et al. (2024) investigated the impact of differentiated instructional practices (DIP) on the academic performance of 300 grade 12 students in the Philippines. Using a quasi-experimental design with pretest/post-test measures, data collected through standardized assessments indicated that DIP significantly improved performance ($M = 90$) compared to traditional methods ($M = 78$), with a p-value $< .05$. The results acknowledge the importance of structuring instruction to meet diverse learner needs,

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aligning with the principles of the 8Ps learning model offering a structured plan for differentiated instruction, promoting flexibility and responsiveness in teaching practices to enhance student engagement and success.

Purnomo et al. (2024) evaluated into how Indonesian undergraduate students approach problem solving when faced with higher-order thinking skills (HOTS)-based calculus questions. Their quantitative analysis of test data yielded significant differences in students' problem-solving skills, often linked to insufficient conceptual understanding of calculus. The authors proposed incorporating problem-solving tasks that can challenge students to apply their knowledge to complex situations. Thus, adopting the 8Ps problem-solving model may assist grade 12 students enhance conceptual understanding and application skills, particularly for solving problems related to stationary points differential calculus.

A mixed-method study conducted in Malaysia by Abd-Hamid et al. (2019) inquired into students' use of graphs to understand derivatives, revealing that while students could perform calculations, they struggled to interpret the derivative concept graphically. The study utilized surveys, tests, and classroom observations to identify the disconnect between symbolic and graphical representations. Findings suggested that teaching should focus more on connecting these representations. In the context of the current study, integrating graphical representations into the 8Ps Learning Model will likely support students in visualizing and understanding stationary points.

Essentially, this literature review explores the ongoing challenges in teaching differential calculus and related mathematical concepts, particularly regarding students' difficulties with conceptual and procedural understanding. Major implications for instructional improvement through the 8Ps learning model are noted. There is a need for interventions within the 8Ps model to address misconceptions about differential calculus (particularly stationary points), and their graphical representations, which hinder deep understanding. It is also important to integrate graph-based and visual activities, as this can bridge the gap between symbolic calculus concepts and their graphical interpretations, fostering meaningful connections. Besides, incorporating problem-solving tasks and interactive experiences within the 8Ps framework can stimulate higher-order thinking skills and enhance conceptual understanding of calculus principles.

Moreover, structured feedback mechanisms in the 8Ps learning model can cater to individual student needs, addressing diverse learning challenges and improving overall learning outcomes. Summarily, adopting the 8Ps learning model offers a promising approach to overcoming instructional challenges by focusing on misconceptions, leveraging visual learning, promoting active engagement, and providing tailored feedback. In all, even though previous studies have shed light on differential calculus instruction for grade 12 students in South Africa, there is limited discussion on the use of student-oriented instructional methods specifically for stationary points. This underscores the essence of the present study, which introduces a novel learning model aimed at enhancing student engagement and performance in stationary points differential calculus and broader mathematics curriculum.

Methodology

Study Design

This quantitative research adopted the quasi-experimental design of the non-equivalent, pre-test/post-test control group to analyze the differences in the test mean scores between experimental and control groups. As such, it was able to measure the effect of the manipulated variable (8Ps instruction) on the dependent variable (student performance in stationary points differential calculus). The group identified as experimental in this study was exposed to the 8Ps treatment while the one which received the traditional instruction was the control group. Summarized as follows in Figure 1 is the study design.

Table 1. Quasi-Experimental Design of the Non-Equivalent, Pre-Test/Post-Test Control Group

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Experimental Group	O_1	X_1	O_2
Control Group	O_1	X_2	O_2

Table 1 shows that, before the intervention, both groups wrote the pre-test (**O_1**) based on stationary points differential calculus. Thereafter, the experimental group was exposed to 8Ps-based treatment (**X_1**) on the mathematical concept. The control group received traditional instruction (**X_2**) on the same leaning content, without partaking in the intervention. Later, both groups wrote the post-test (**O_2**).

Sampling

The target population comprised grade 12 mathematics students and teachers within the Tshwane West education district, Gauteng province. Eight schools were purposively selected for the study. Andrade (2021) notes that purposive sampling is advantageous because it considers participants with attributes suited for the research goals. The eight schools chosen had comparable geographical locations, teacher qualifications, infrastructure, student performance, instructional resources and English as the language of instruction. This similarity ascertained that any observed differences in outcomes would likely reflect the effects of the intervention rather than pre-existing disparities (Creswell, 2021). Four schools, designated as the experimental group, consisted of 128 students and 4 teachers. The remaining four schools, which formed the control group, had 125 students and 4 teachers. Altogether, the study sample contained 253 students and 8 teachers.

To forestall any potential contamination of research results, the four schools in the experimental group were situated at a reasonable distance from the four schools in the control group. This separation was to ensure that the control group did not gain access to the 8Ps intervention (Singh, et al. 2021). Participants were not randomly assigned as the study involved one intact grade 12 mathematics class per participating school, a decision taken to maintain the natural classroom environments and minimize disruptions (Creswell, 2021). Moreover, the teachers engaged had similar qualifications (minimum of a bachelor's degree in education with about ten years of grade 12 mathematics teaching experience) to enhance internal validity and minimize potential bias (Dhlamini, 2012; Jacob, et al., 2017). Basically, the DoE, South Africa, requires a high school teacher to hold a Bachelor of Education or a Postgraduate Certificate in Education (PGCE) (Mthethwa & Mkhize, 2021; Ndlovu & Moyo, 2023).

Instrumentation and Validation

The research employed a mathematics achievement test as the primary tool for data collection. This test was based on the 2017-2019 DBE NSC examination questions specifically focused on the concept of stationary points in differential calculus (see the Appendix). It was administered both as pre-test and post-test. Even though the questions adhered to the grade 12 CAPS mathematics curriculum and the assessment criteria prescribed by Umalusi (Council for Quality Assurance in General and Further Education and Training), they still underwent standard procedures for ensuring validity and reliability. The selected questions required participants to apply various mathematical reasoning skills and solution strategies, covering all aspects of stationary points specified in the curriculum. As outlined by DBE (2012), these aspects are: minima, maxima and points of inflections; the factor theorem and other useful techniques for determining the x-intercepts, and practical problems related to rates of change and optimization. The five test-items had point-biserial correlation coefficients ranging from 0.3 to 0.58, which according to Fadilah (2014), demonstrates a reasonable degree of item discrimination for a valid test.

To verify the quality of the test, two instrument development specialists attended to its technical details, while four experts in mathematics education – three experienced grade 12 teachers with close to two

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decades in the field and a Subject Advisor from a district education office – carefully evaluated it for face and content validity. The test was also pilot-tested on 82 students from a high school outside those ones for the main study. Using a test-retest method, the test was administered twice over a two-week interval, yielding a correlation coefficient of 0.87, reflecting strong reliability. For internal consistency, the test attained a Cronbach's alpha value of 0,698, a value close to the generally acceptable threshold of 0.7, confirming adequate reliability in terms of internal consistency (Badenes-Ribera, et al., 2024). The final version of the test eventually emerged with five questions with each one having sub-questions.

Ethical Measures

In adherence to research ethics (see (Creswell, 2021; Thomas & Rothman, 2013), this study was executed in accordance with established standards, norms and practices. Approval was secured from all relevant authorities prior to starting. The purpose and benefits of the research were clearly communicated to the participating schools, teachers and students, who all gave their consents. They were assured of their safety, informed that participation was voluntary, and reminded they could withdraw at any time of the study without consequences. The anonymity and confidentiality of any information they supplied were guaranteed. Both students and their parents signed the informed consent forms to confirm their participation.

Classroom Implementation of the 8Ps Intervention

The researcher personally implemented the intervention in the experimental group, with the regular mathematics teachers as lesson observers. This strategy was designed to ensure consistent and thorough application of the 8Ps learning model, maximizing its potential effect. Cresswell (2021) points out that direct involvement allows the researcher to monitor and adapt the intervention to suit participants' needs, enhancing its relevance and effectiveness within the study's context. Dhlamini (2012) notes that this hands-on approach aids in understanding participants' learning challenges and fosters trust, a critical factor in successful educational interventions. Similarly, Masilo (2018) submits that personal delivery maintains the intervention's fidelity and quality control, while Ofori-Kusi (2017) highlights that it offers deeper insight into participant engagement and responses, facilitating necessary adjustments to improve outcomes. To reduce potential biases, the researcher relied on previous studies and literature to guide his role.

In the control group's schools, the regular mathematics teachers taught the same content on stationary points in differential calculus using the traditional methods. The researcher visited their classes as a non-participant observer, attending four lessons per school. From the observations, it was noted that the traditional approaches those teachers applied were different from the 8Ps learning model. He noticed that the students sat in rows, mostly listening to the teacher-led, whole-class instruction. Most times, the teachers dominated the learning processes, allowing minimal student interaction. In contrast, the experimental group had students seated in small, mixed-ability groups of four to five students each. The researcher monitored and facilitated the groups as they worked through the 8Ps learning process in solving questions assigned to them, encouraging interactions and discussions. Both groups completed the pre-test at the beginning of the intervention to assess their prior knowledge of stationary points, and the post-test at the end to measure possible improvement in their performances. The researcher administered both tests with assistance from the teachers. For anonymity, students were assigned unique codes to write on their test scripts instead of their names. Following the two-month investigation, the tests were marked, and the results documented for analysis. Figure 3 below shows the data-gathering procedure adopted.

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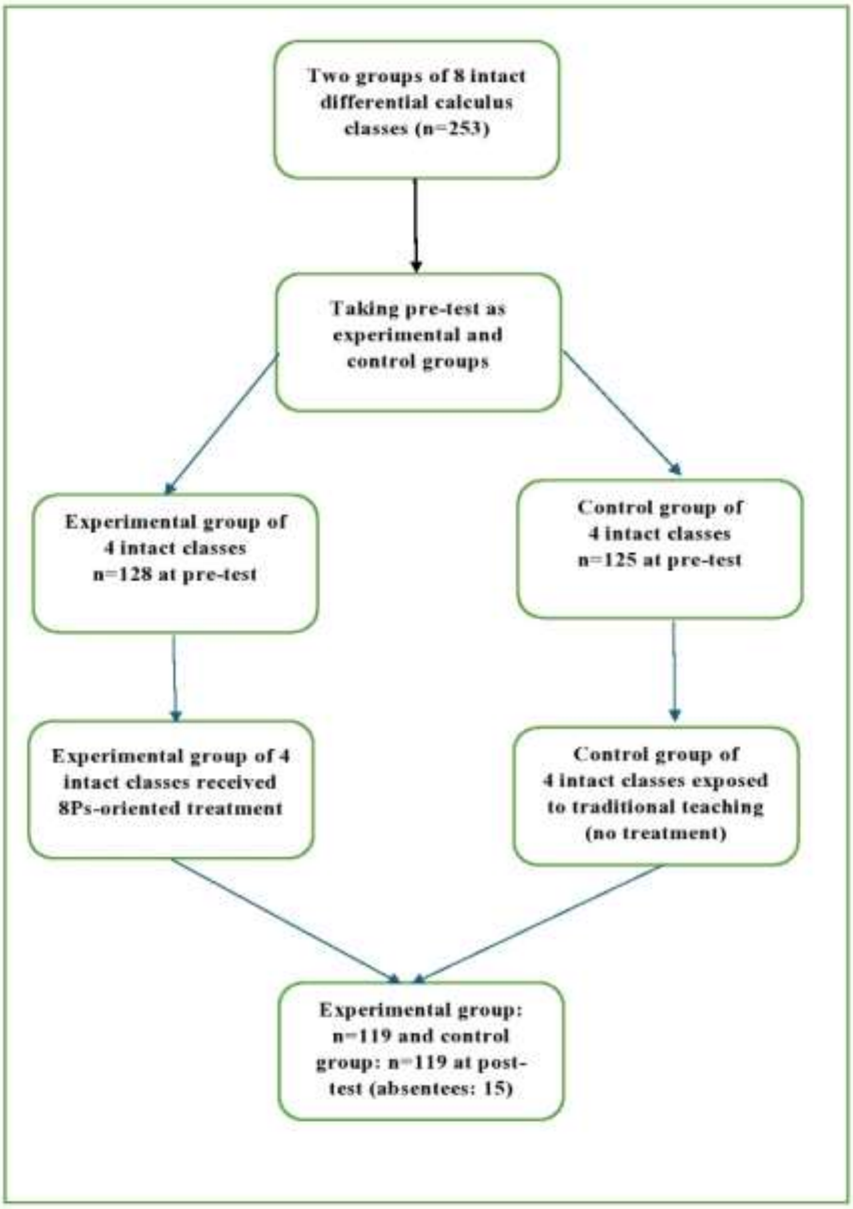
Figure 3. Flow Chart of the Data-Collection Process



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Practical Application of the 8Ps Learning Model that Each Intervention Lesson Followed

In this section, we explain how the intervention teacher utilized the 8Ps learning model in teaching the concept to the participants in the experimental group, to demonstrate how the model can be applied in practice to teach a mathematics lesson.

Problem: Calculate the coordinates of the turning points of $f(x) = 2x^3 - 5x^2 + 4x$ and draw its graph.

Phase 1 (Probing): This stage requires a critical examination of the question. The problem solver can do this by posing and answering questions such as: (i) What type of function is given in the question? *It is a cubic function*; (ii) What does the question particularly call for? *It asks for the turning points and the graph of* f; (iii) What is a turning point of a graph? *It is a point on the graph where the direction of the graph changes, either from increasing to decreasing or vice versa*; (iv) Do the coordinates of a turning point refer to only the y-coordinate? *No, it refers to both the x-coordinate and the y-coordinate*; (v) How are the coordinates of a turning point correctly written? *They are written as* $(x; y)$; Is/are there any other action(s) to take to be able to draw the required graph of f? *Yes, in addition to finding the turning points, we need to calculate the* x*-intercepts and* y*-intercept of* f, etc.

Phase 2 (Pinpointing): Pointing out or recognizing the specific terms, variables and conditions in the question is crucial for gaining valuable insights into the problem. The key words in this question are *calculate, coordinate*s, *turning points* and *draw*. In relation to the key words pinpointed, the problem solver can engage in a form of reasoning to determine the appropriate steps to follow in solving the problem. For example, the question says *calculate*, which implies that the solver must perform precise mathematics computations and show all the solution steps, rather than estimate, guess, or simply write down the answer. Again, the question specifically asks for turning points, not x- and y-intercepts, which some problem solvers may mistakenly assume. It requires the coordinates of the turning points, meaning both x-coordinate and y-coordinate for each turning point must be found, not the y-coordinate alone. The question also demands that the graph of f be drawn, not merely sketched. This shows that all the values needed for producing the graph must be accurately calculated and plotted.

Phase 3 (Patterning): To have a deeper understanding of how to successfully solve the problem, the problem solver should explore whether the question can be expressed as helpful patterns. Transforming the question into an equation, or using it to create a graph, table, chart, illustration, map, diagram or any other relevant representations may uncover valuable clues leading to effective solution strategies. For instance, the problem solver can infer the following points from the problem as shown in Table 2:

Table 2. A Helpful Mathematical Pattern for the Given Problem

Valuable clues	$f(x) = 2x^3 - 5x^2 + 4\text{x}$
Degree of f	3
Function type	cubic
Factorized form	$(2x - a)(x - a)$
Number of x-intercepts	3
Number of y-intercepts	1

Since the question pertains to a cubic function of the general form $ax^3 - bx^2 + \text{cx} + \text{d}$, it must have two turning points – one which is a local (or relative) maximum and the other, a local (or relative) minimum. The characteristics of these two turning points can be analyzed by their concavity as illustrated in Table 3 below:

Table 3. Another Helpful Mathematical Pattern for the Given Problem

Minimum turning point	**Maximum turning point**
Concave downwards (the curve's gradient is moving from increasing to decreasing)	Concave upwards (the curve's gradient is moving from decreasing to increasing)

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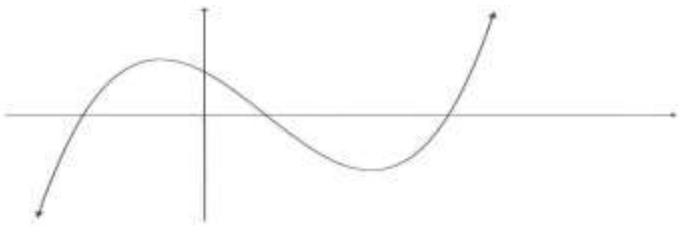
U shape	∩ shape
$f''(x) > 0$	$f''(x) < 0$
$f'(x) = 0$	$f'(x) = 0$

Although the problem solver is expected to draw the graph of f, it can be helpful to first sketch a preliminary version as a reference for the final drawing. The value of constant a plays a vital role in determining the shape of the graph of f. For $2x^3 - 5x^2 + 4x = ax^3 - bx^2 + cx + d, a > 0$. The graph is then sketched as follows:

Figure 4. *A Graph Sketch of the Function* $f(x) = 2x^3 - 5x^2 + 4x$ *as a Helpful Pattern*



Phase 4 (Projecting): The problem solver needs to formulate effective solution strategies at this point. Drawing from the mathematical reasoning established in phases 1 through 3, they must select the appropriate mathematical operations, make necessary assumptions, and outline the strategies and procedures required to tackle the problem at hand. Here are some viable solution approaches for the current question:

Step 1: Find the derivative, $f'(x)$, of the function $f(x) = ax^n$, applying the general differentiation rule expressed as: $f'(x) = \frac{dy}{dx} = anx^{n-1} = 0$. This equation holds true because, at a turning or stationary point, the slope of the curve (i.e., the derivative) is always zero.

Step 2: Obtain the zeroes or factors of $f'(x)$. These zeroes correspond to the x-coordinates of the turning point. Use one of the following three formulas: factorization, the general method $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ or the completing the square method.

Step 3: To calculate the value of *y* (or the *y*-coordinate) for each *x,* substitute each *x* value into $y = f(x)$. The resulting coordinates of the turning points will be expressed as $(x; y)$.

Step 4: Determine the *x*- and *y*-intercepts using the following principle: At *x*-intercept, y = 0, and at *y*-intercept, x = 0.

Step 5: Calculate the *x*-intercept, applying either of $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ or $x = b^2 - 4ac$.

Phase 5 (Prioritizing): This phase demands that the potential solution ideas be evaluated and prioritized based on their relevance and effectiveness. By arranging the projected solution plans in order of their usefulness, the problem solver can focus their efforts on the most suitable options while eliminating the less-important ones that are not directly related to the question. In this case where $f'(x)$= 0 is factorizable, it is reasonable to prioritize the factorization method to obtain the *x*-coordinates. This approach is usually more efficient compared to the general method or the completing the square method, which can be time-consuming in such situations. Moreover, this same principle of prioritizing can enable the problem solver to think of obtaining the *x*-intercept through the formula $x = b^2 - 4ac$, because it is quicker and easier to use than $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$.

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Phase 6 (Processing): Here, the already prioritized solution strategies are utilized to solve the question as shown below: $f(x) = 2x^3 - 5x^2 + 4x$

$f'(x) = 6x^2 - 10x + 4 = 0$ [Set $f'(x) = 0$, since gradient of a curve = 0 at a turning point].

Divide through by 2, we have $3x^2 - 5x + 2 = 0$.

Factorizing, we have, $x = \frac{2}{3}$ or $x = 1$ (x-coordinates)

For *y*-coordinates, substitute each *x* value in $y = f(x) = 2x^3 - 5x^2 + 4x$

When $x = \frac{2}{3}$, $y = 2(\frac{2}{3})^3 - 5(\frac{2}{3})^2 + 4(\frac{2}{3}) = \frac{28}{27}$

Also, when $x = 1$, $y = 2(1)^3 - 5(1)^2 + 4(1) = 1$

Turning points = $(\frac{2}{3}; \frac{28}{27})$ and (1; 1)

Now, the intercepts are calculated as follows: At *y*-intercept, $x = 0$.

$y = f(x) = 2x^3 - 5x^2 + 4x = 2(0)^3 - 5(0)^2 + 4(0) = 0$. Thus, *y*-intercept = 0

At x-intercept, $y = 0$, we have $2x^3 - 5x^2 + 4x = 0$ yielding $x(2x^2 - 5x + 4) = 0$.

Therefore, $x = 0$ or $x = b^2 - 4ac$ (where $a = 2,\ b = -5$ and $c = 4$)

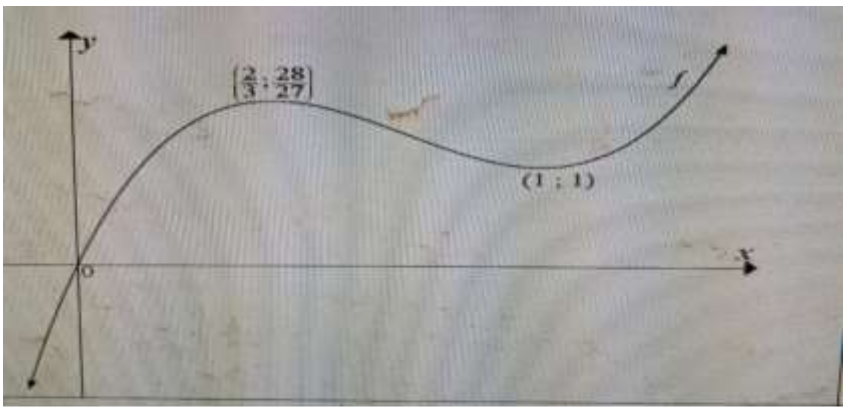
$x = 0$ or $x = (-5)^2 - 4(2)(4) = 25 - 32 = = -7$

$x = 0$ or $x = -7$ (*x*-intercepts)

With the values of the two turning points, x-intercepts and y-intercept, the graph of $f(x)$ is then accurately drawn as Figure 5 below:

Figure 5

The Accurate Graph of the Function $f(x) = 2x^3 - 5x^2 + 4x$



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Phase 7: (Proving): Once the mathematics problem has been solved, this phase requires a reflection on the solution to be sure it is correct. For the assurance of this, the problem solver must review the strategies that produce the solution. They can do this as shown below:

First, check that $f(x) = 2x^3 - 5x^2 + 4x$ is correctly differentiated.

Then, substitute each of $x = \frac{2}{3}$ and $x = 1$ in $f'(x) = 6x^2 - 10x + 4$ to check whether it will yield zero since $f'(x) = 0$ at a turning point. This is done as follows:

When $x = \frac{2}{3}$, $f'(x) = 6x^2 - 10x + 4 = 6(\frac{2}{3})^2 - 10(\frac{2}{3}) + 4 = \frac{24}{3} - \frac{20}{3} + 4 = 0$

When $x = 1$, $f'(x) = 6(1)^2 - 10(1) + 4 = 6 - 10 + 4 = 0$

He can as well verify the calculations for the *y*-coordinates, x- and y-intercepts, and confirm that the values are accurately plotted to create the desired graph of f.

Phase 8 (Predicting): In this final phase, the problem solver is tasked with determining whether the solution can be generalized or applied to similar mathematics tasks. They must assess whether the solution can be utilized to predict or derive the solutions to other related mathematics questions. This phase ultimately defines the level of acceptability of the solution. In this case, the above solution to $f(x) = 2x^3 - 5x^2 + 4x$ can be used to predict the solution to a similar function, $g(x) = x^3 - x^2 - x + 1$. Since $g(x)$ is also a cubic function, it will exhibit one y-intercept, three x-intercepts and two turning points (one representing a local maximum and the other a local minimum). In addition, the equation $ax^3 - bx^2 + cx + d = x^3 - x^2 - x + 1$ indicates that $a > 0$, suggesting that the graph of $g(x)$ will share a similar shape with that of $f(x)$. Conversely, consider the function $h(x) = -5x^3 + 3x^2 - 2x - 1$, where $a < 0$. The graph of this function will have a different shape. It is important to note that not all mathematical problems will necessitate the application of all eight steps of the learning model.

Results

The pre-test and post-test marks for the two groups were analyzed through descriptive statistics and the independent samples t-test to provide insightful information about how the implementation of the 8Ps learning model had impacted the students' performance in stationary points differential calculus. The process began with comparing the descriptive statistics of both groups' marks to establish a basis for the subsequent independent samples t-test meant for determining the statistical significance of any observed differences. It is worth noting that 15 (experimental group = 9 and control group = 6) of the 253 participants who wrote the pre-test could not make it to the post-test stage. Hence, the data analysis was based on 238 participants (experimental group =119 and control group = 119). Justifying the exclusion of the 15 participants who missed the post-test, Shields (2019) contends that this action ensures valid statistical analysis and reduces confounding variables that can skew the result, which ultimately makes the research outcomes more reliable.

Descriptive Statistics for Both Groups' Pre-test Marks

The descriptive statistics of the marks obtained by both groups are compared as follows:

Table 4. Comparing the Descriptive Statistics for Pre-test Marks of Experimental and Control Groups

Pre-test	n	M	Median	SD	Min	Max	Range	Q_1	Q_3	Coeff. of Var.
Exp. Grp.	119	5.1	5	3.36	0	12	12	3	7	65.89 %

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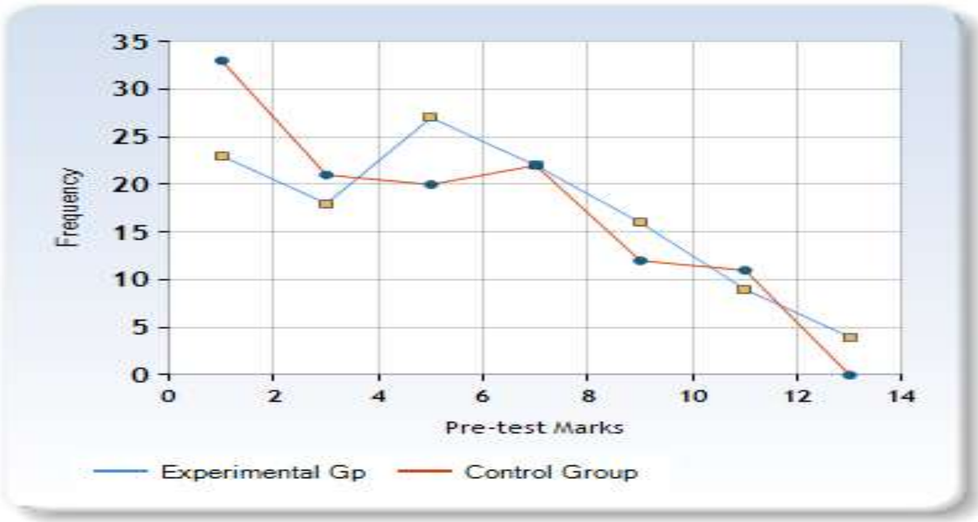
Cont. Grp.	119	4.39	4	3.34	0	11	11	1	7	76.14 %

According to Table 4, the results of the experimental group (M = 5.1; SD = 3.36) and control group (M = 4.39; SD = 3.34) exhibited comparable pre-test knowledge levels regarding stationary points in differential calculus. Supporting this observation, Table 3 reveals that their mean and median scores are relatively close. Additionally, their closely matched median, quartile and range values also reflect a similar distribution of scores across both groups. This is further illustrated by Figure 6 below.

Figure 6. Pre-test Frequency Polygons of Both Groups



The overlapping frequency polygons depicted by Figure 6 indicate that both experimental and control groups had similar distributions of marks prior to the intervention, with no initial advantage to either group. This observation suggests that they entered the investigation with comparable levels of knowledge about stationary points differential calculus, thereby creating a fair baseline for assessing the impact of the 8Ps learning model.

Independent Samples T-test of Both Groups' Pre-test Marks

To further ascertain that both groups encountered this research at an equivalent baseline, the independent samples t-test was performed α = .05 level of significance to find out whether their mean difference is statistically significant. Table 5 below summarizes the results.

Table 5. A Summary of Independent Samples T-test of Pre-test Marks of Experimental and Control Groups

Test	Group	n	M	SD	df.	t	P	Cohen's d
Pre-test	Exp. Grp.	119	5.1	3.36	118	1.64	.1	0.21
Pre-test	Cont. Grp.	119	4.39	3.34	118			

From Table 5, the t-test result, {t(118) = 1.64, p = .1, p > α = .05, two-tailed}, indicates that there is no statistically significant difference between the mean scores of the experimental group ($M = 5.1,\ SD = 3.36$) and the control group ($M = 4.39,\ SD = 3.34$). with a low Cohen's d effect size of 0.21. This finding suggests that both groups were equivalent at the start of the study. The overlapping frequency polygons of both groups also lend credence to their equivalence at pre-test stage.

Descriptive Statistics for Both Groups' Post-test Marks

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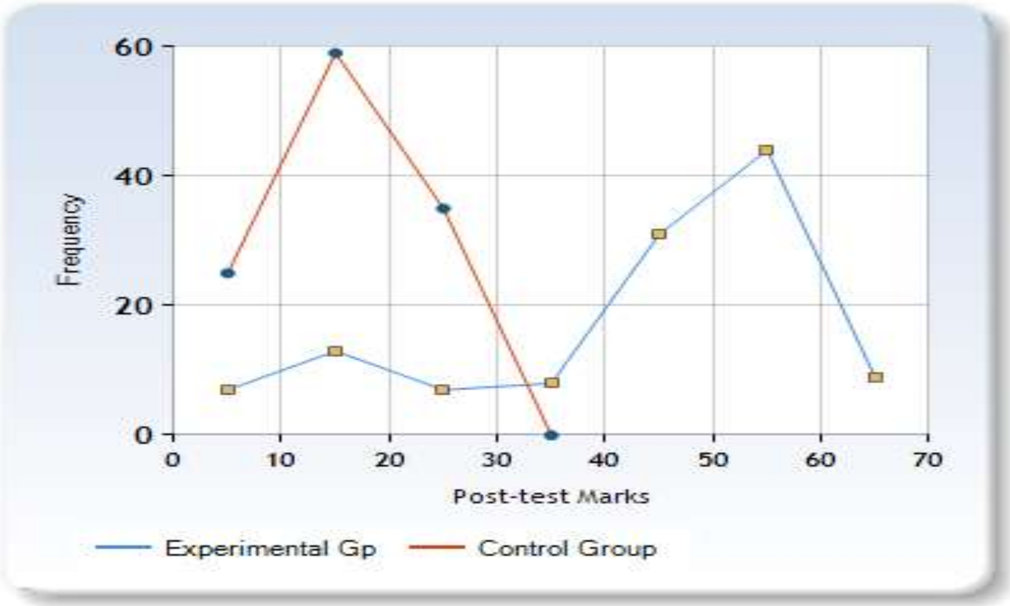
The descriptive statistics for the experimental and control groups were also compared to evaluate their levels of improvement in the post-test as shown by Table 6.

Table 6. Comparing the Descriptive Statistics for Post-test Marks of Experimental and Control Groups

Post-test	n	M	Median	SD	Min	Max	Range	Q_1	Q_3	Coeff. of Var
Exp. Grp.	119	41.98	48	16.28	6	62	56	33	55	38.79%
Cont. Grp.	119	16.01	16	6.15	3	29	26	10	21	38.39%

Table 6 reveals that the experimental group achieved (M = 41.98; SD = 16.28) while the control group obtained (M = 16.01; SD = 6.15) in the post-test. This predicts that the experimental group outperformed the control group with a mean score difference of 25.97%. The median, range and standard deviation values recorded here are also pointers to the group differences. This is also further revealed by Figure 7 below.

Figure 7. Post-test Frequency Polygons of Both Groups



In Figure 7, there is no overlap between the two frequency polygons as evidenced by the noticeable rightward shift of the experimental group's polygon following the intervention. The gap between the polygons here signifies a higher improvement for the experimental group, with a greater number of participants achieving better marks.

Independent Samples T-test of Both Groups' Post-test Marks

To determine whether the 25.97% mean score increase that the experimental group attained above the control group is statistically significant, the independent samples t-test of both groups' post-test marks was run at a = .05 significance level. The motive was to probe the null hypothesis stating that there is no statistically significant quantitative difference between the mathematical problem-solving performances of students taught with the 8Ps learning model and those instructed through the traditional teaching methods. Table 7 below captures the outcome of the t-test.

Table 7. A Summary of Independent Samples T-test of Post-test Marks of Experimental and Control Groups

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Test	Group	n	M	SD	df.	t	P	Cohen's d
Post-test	Exp. Grp.	119	41.98	16.28	118	16.28	$p < .00001$	2.11
Post-test	Cont. Grp.	119	16.01	6.15	118			

From Table 7, the t-test result, {t(118) = 16.28, p < .00001, p < α = .05, two-tailed} unveils that the 25.97% mean difference between the experimental group (M $= 41.98,\ SD = 16.28$) and the control group (M $= 16.01,\ SD = 6.15$). is statistically significant. With a 95% confidence interval, the mean increase of 25.97% falls within 22.83 and 29.11. A large Cohen's d effect size of 2.11 is produced.

Discussion of Results

Research Question: *Will the implementation of the 8Ps learning model in teaching and learning stationary points in differential calculus have any statistically significant quantitative effect on grade 12 students' performance?*

This study sought to quantify how implementing the 8Ps learning model would affect the performance of grade 12 students in stationary points differential calculus. As such, employing a quasi-experimental design with pre-test/post-test, non-equivalent control group, the study raised and addressed the afore-stated question. The pre-test and post-test marks recorded by both experimental and control groups were analyzed via descriptive statistics and independent samples t-test. Pre-test results indicated that both the experimental group (M $= 5.1,\ SD = 3.36$) and control group (M $= 4.39,\ SD = 3.34$) exhibited comparable baseline knowledge of stationary points differential calculus (see Table 4). The closely matched descriptive statistics (mean, median, range, quartiles and standard deviation) attained by both groups suggest this (Shields, 2019). As illustrated by Figure 6, the comparability is further highlighted by their overlapping frequency polygons, indicating that, before the intervention, there were similar distributions of marks across both groups (Kim (2015). Also acknowledging the utility of frequency polygons as an invaluable analytical research tool, Zhao (2019) posits that they facilitate effective comparisons between control and experimental groups, deepening insights into data distributions, aiding hypothesis testing, and revealing outliers or anomalies within datasets. This reflects similar pre-intervention conditions for both groups, thus reinforcing the credibility of evaluations regarding the effects of the intervention.

In consistency with Kim (2015), despite obtaining these findings that were suggestive of group equivalence, we considered it important to perform an independent samples t-test to account for biases that might arise from the non-random sample selection. This t-test also served to verify that both groups were statistically similar on the pre-test measures. Kim further declares that validating the statistical similarity of both groups with the t-test is crucial for strengthening a study's internal validity. In a related observation, Mills & Gay (2016) in case of any statistically significant differences at this point, the researcher may have to adjust their analyses or consider using an alternative research design. The writers also note that conducting a t-test at this point was vital to confirm that any post-intervention changes were genuinely traceable to the treatment rather than pre-existing knowledge disparities – a critical consideration in educational research where prior knowledge can greatly affect learning outcomes. The results from the t-test {t(118) = 1.64, p = .1, p > α = .05, two-tailed} showed a low Cohen's d effect size of 0.21, indicating no statistically significant mean difference between groups (see Table 4). These analytical steps, collectively, suggest that any differences noticed after the intervention might stem from the effects of the 8Ps treatment rather than the initial knowledge disparities, thereby bolstering the internal validity and conclusions of the study (Kim, 2015).

The results from the post-test analysis show that the experimental group attained a mean score increase of 25.97% above the control group after receiving the 8Ps treatment. The experimental group's standard deviation 16.28 indicates a wide spread of scores away from the mean, while the control group's standard deviation = 6.15 unveils scores clustering around the mean (see Table 5). Interpreting this, Shields (2019) states that score variability can influence instructional strategy effectiveness, implying that diverse learning experiences lead to varied levels of student performance. The median scores for both groups – 48 for the experimental group and 16 for the control group – indicate that half of each group's participants scored at least these values. Furthermore, while the experimental group achieved a score range of 6 - 62 marks, the

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control group only made a score range of 3 - 29 marks (refer to Table 5). This disparity suggests that improvements were more pronounced among the participants in the experimental group relative to their control group mates. Shields further comments that median and range scores can provide useful insights into the overall group performance disparities brought about by different instructional approaches.

Moreover, the post-test results reveal variability in score distribution between both groups, evidenced by a rightward shift in the frequency polygon for the experimental group (see Figure 7). Zhao, et al. (2019) describe the shift as a considerable improvement in performance of the experimental group since more participants in the group recorded higher marks after the intervention. The improvement aligns with the findings from Nuñez et al. (2023) who obtained a similar result by applying Enhancing Mastery & Expertise in Mathematics (EM&EM) instruction to assess the academic performance of 46 grade 11 students in basic calculus concepts. In addition, the independent samples t-test result $\{t = 16.28, p < .00001, p < \alpha = .05$; two-tailed$\}$ confirms that the 25.97% mean difference is statistically significant (see Table 6), reinforcing Zhao (2019) who posits that educational interventions can produce significant positive outcomes especially when properly implemented. Besides, a Cohen's d of 2.11 signifies a large effect size (see Table 6), suggesting that the mean difference is considerable relative to the score variability within each group and highlights a strong impact of implementing the 8Ps learning model. In support, Lovakov and Agadullina (2021) reiterate that effect sizes are essential for understanding how educational interventions influence learning outcomes, further buttressing our conclusion about the positive effects of the 8Ps treatment on the student mathematics performance.

Consequently, we reject our initial null hypothesis stated as*: There is no statistically significant quantitative difference between the academic performances of the participants in the experimental group exposed to the 8Ps instruction and their counterparts in the control group taught with the traditional methods.* We then declare that the experimental group which instructed through the 8Ps learning model achieved a statistically significant mean difference of 25.97% over the control group that received traditional instruction on the same content of stationary points differential calculus. By that, the research question has been answered.

Study Limitations

This research, like many others, has limitations that could impact on its overall generalizability. The quasi-experimental design, with pre-test/post-test, non-equivalent control group, might introduce biases. Even though some measures to ascertain group comparability were taken, the absence of random assignment means there could still be pre-existing differences between the two groups, which might affect the results. The non-randomization restricts the ability to draw firm causal claims concerning the effectiveness of the 8Ps learning model compared to traditional teaching methods. Besides, the sample of 253 from eight high schools may not accurately reflect the broader population of grade 12 mathematics students, potentially limiting the external validity of the findings. Furthermore, using pre- and post-test measurements might have caused testing effects, with participants improving in the post-test not necessarily because of the 8Ps treatment, but because they were familiar with the test format. Each of these limitations could independently influence the results regarding the effectiveness of the 8Ps learning model.

Conclusion

This study has demonstrated how the 8Ps learning model can be effectively integrated into mathematics instruction, focusing on stationary points differential calculus in grade 12. The findings suggest a statistically significant mathematics performance among the students who experienced the 8Ps-oriented instruction compared to those instructed through traditional methods, reinforcing the need for innovative pedagogical strategies in the mathematics classroom. Therefore, if thoughtfully implemented, the 8Ps learning model can be a practical guide for teachers and policymakers aiming to develop students' problem-solving skills and elevate mathematics education. Nevertheless, given the study's limitations which may affect the wider applicability of these findings, further research is advised to test the efficacy of the model in various educational settings. Continued investigation and adaptation of the 8Ps model is crucial in advancing mathematics education and ensuring that students are equipped with essential mathematical problem-solving skills for success in a rapidly evolving world.

Contribution to the Literature

- The study offers solid empirical evidence for the 8Ps model's capacity to improve grade 12 student mathematics performance, particularly in stationary points differential calculus.

- The study reviews prominent previous learning models, highlighting their fundamental principles and mechanisms, and presenting them as valuable instructional strategies for improving mathematics education.

- The study draws on Dewey's theory of reflective enquiry and experiential learning as a basis for designing and implementing the 8Ps learning model, stressing the relevance of Dewey's ideas in contemporary education while contextualizing the 8Ps learning model.

- The study's findings bear far-reaching implications for educational policy and practice, both in South Africa and worldwide. By constructing the 8Ps learning model, showcasing its practical use for mathematics instruction, and proving its potential to enhance student performance, especially in grade 12 differential calculus, the study provides insightful guidance for education policy makers, teachers and students, helping them shape informed decision around teaching techniques and problem-solving approaches.

Ethical clearance (2018_CGS/ISTE+006) for this study was sought and obtained from the Unisa ISTE Ethics Review Committee prior to the conduct of the study.

Informed Consents: Participants' consents were sought and obtained.

Data-sharing statement: The dataset for this study can be made available by the Corresponding Author upon a reasonable request.

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Conflict of Interests: We have no conflict of interests to disclose

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LIST OF APPENDICES

Appendix A1: Pre-test

Learner's Code: ………… Learner's Class: A ☐ B ☐ (Tick as applicable)

Instruction: Answer all the questions showing clearly all your calculations.

Duration:1 hour 30 minutes

QUESTION 1 (Feb/March 2017 Q8)

Given: $f(x) = 2x^3 - 5x^2 + 4x$

1.1	Calculate the coordinates of the turning points of the graph of f	(5)
1.2	Prove that the equation $f(x) = 2x^3 - 5x^2 + 4x$ has only one root	(3)
1.3	Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points	(3)
1.4	For which values of x will the graph be concave up?	(3)
		[14]

QUESTION 2 (May-June 2017 Q9)

Given: $f(x) = x^3 - x^2 - x + 1$.

2.1	Write down the coordinates of the y-intercepts of f.	(1)
2.2	Calculate the coordinates of the x-intercepts of f.	(5)
2.3	Calculate the coordinates of the turning points of f	(6)
2.4	Sketch the graph of f.clearly indicating all intercepts with the axes and the turning points.	(3)
2.5	Write down the values for which $f''(x) < 0$.	(2)
		[17]

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4.3 Solve $f(x) = f'(x)$. (3)

4.4 The graphs f , f' and f'' all pass through the point (0; 0).

For which of the graphs will (0; 0) be a stationary point? (1)

4.5 Explain the difference, if any, in the stationary points mentioned in (4.3.2) (2)

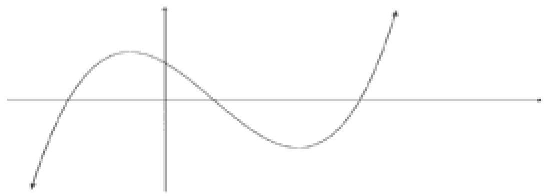
4.6 Determine the vertical distance between the graphs of f' and f'' at $x = 1$ (3)

[17]

QUESTION 5 (Feb/March 2018 Q9)

The sketch below represents the curve of $f(x) = x^3 - bx^2 + cx + d$.

The solutions of the equation $f(x) = 0$ are -2 ; 1 and 4



5.1 Calculate the values of b, c and d. (4)

5.2 Calculate the coordinates of B, the maximum turning point of f. (4)

5.3 Determine an equation for the tangent to the graph of f at $x = -1$. (4)

5.4 Sketch the graph of $f''(x)$ and clearly indicate the x- and y-intercepts. (3)

5.5 For which value(s) of x is $f(x)$ concave upwards? (2)

[17]

TOTAL: 85

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QUESTION 3

3.1 The function $f(x) = x^3 + bx^2 + cx - 4$ has a point of inflection at $(2 ; 4)$. Calculate the values of b and c (May-June 2017 Q8.3). (7)

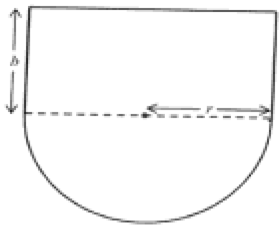
3.2 Given: $f(x) = -3x^3 + x$. Calculate the value of q for which $f(x) + q$ will have a maximum value of $\frac{8}{9}$ (Feb/March 2018 Q10). (6)

3.3 A piece of wire $6\ m$ long is cut into two pieces. One piece, $x\ m$ long, is bent to form a square ABCD. The other piece is bent into a U-shape to form a rectangle BEFC when placed next to the square.

Calculate the value of x for which the sum of the areas enclosed by the wire will be a maximum (Feb/March 2017 Q9). (7)

[20]

QUESTION 4 (May-June 2017 Q10)



The figure is a design of a theatre stage in the shape of a semi-circle attached to a rectangle. The semi-circle has a radius r and the rectangle has a breadth b. The perimeter of the stage is $60\ m$.

4.1 Determine the expression for b in terms of r. (2)

4.2 For which value(s) of r will the area of the stage be a maximum? (6)

Given that $f(x) = 3x^3$. (Nov 2019 Q9)

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Appendix A2: Post-test

Learner's Code: .... Learner's Class: A ☐ B ☐ (Tick as applicable)

Instruction: Answer all the questions showing clearly all your calculations.

Duration: 1 hour 30 minutes

QUESTION 1 (May-June 2017 Q9)

Given: $f(x) = x^3 - x^2 - x + 1$.

1.1	Write down the coordinates of the y-intercepts of f.	(1)
1.2	Calculate the coordinates of the x-intercepts of f.	(5)
1.3	Calculate the coordinates of the turning points of f	(6)
1.4	Sketch the graph of f. Clearly indicate all intercepts with the axes and the turning points.	(3)
1.5	Write down the values for which $f''(x) < 0$.	(2)
		[17]

QUESTION 2 Feb/March 2017 Q8)

Given: $f(x) = 2x^3 - 5x^2 + 4x$

2.1	Calculate the coordinates of the turning points of the graph of f	(5)
2.2	Prove that the equation $f(x) = 2x^3 - 5x^2 + 4x$ has only one root	(3)
2.3	Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points.	(3)
2.4	For which values of x will the graph be concave up?	(3)
		[14]

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4.5 Explain the difference, if any, in the stationary points mentioned in (4.3.2) (2)

4.6 Determine the vertical distance between the graphs of f' and f'' at $x = 1$ (3)

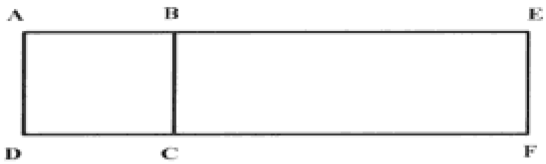
[17]

QUESTION 5

5.1 The function $f(x) = x^3 + bx^2 + cx - 4$ has a point of inflection at $(2 ; 4)$. Calculate the values of b and c. (May-June 2017 Q8.3) (7)

5.2 Given: $f(x) = -3x^3 + x$. Calculate the value of q for which $f(x) + q$ will have a maximum value of $\frac{8}{9}$. (Feb/March 2018 Q10) (6)

5.3 A piece of wire $6\ m$ long is cut into two pieces. One piece, $x\ m$ long, is bent to form a square ABCD. The other piece is bent into a U-shape to form a rectangle BEFC when placed next to the square.



Calculate the value of x for which the sum of the areas enclosed by the wire will be a maximum. (Feb/March 2017 Q9) (7)

[20]

TOTAL: 85

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QUESTION 3 (Feb/March 2018 Q9)

The sketch below represents the curve of $f(x) = x^3 - bx^2 + cx + d$.
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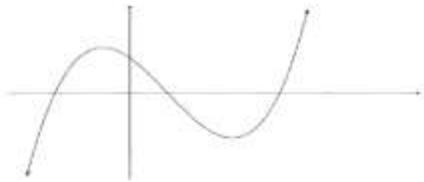
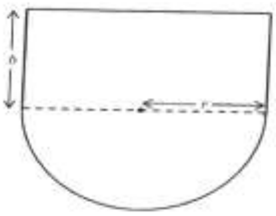


3.1	Calculate the values of b, c and d.	(4)
3.2	Calculate the coordinates of B, the maximum turning point of f.	(4)
3.3	Determine an equation for the tangent to the graph of f at $x = 1$.	(4)
3.4	Sketch the graph of $f''(x)$ and clearly indicate the x- and y-intercepts.	(3)
3.5	For which value(s) of x is $f(x)$ concave upwards?	(2)
		[17]

QUESTION 4 (May-June 2017 Q10)



The figure is a design of a theatre stage in The shape of a semi-circle attached to a rectangle. The semi-circle has a radius r and the rectangle has a breadth b. The perimeter of the stage is $60m$.

4.1	Determine the expression for b in terms of r.	(2)
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4.3	Solve $f(x) = f'(x)$.	(3)
4.4	The graphs f , f' and f'' all pass through the point (0; 0). For which of the graphs will (0; 0) be a stationary point?	(1)